1. Definition of a simple harmonic oscillator.

Answer:
We may define simple harmonic motion (SHM) as,

$$F = -k(x) \text{ - - - - - - - } (i)$$

It is such a motion where the displacement (x) is proportional to the applied force (F) and the displacement is negatively directed to the force.

Where, F = force (N)
K = constant (spring constant)
X = displacement (m)

2. Derive a differential equation for SHM

Answer:

We know that the definition of SHM, $F = -kx$.

We may write $F = ma$ (using Newton's law). So,

$$ma = -kx$$
$$m\,dv/dt = -kx$$
$$m\,d/dt(dx/dt) = -kx$$
$$\mathbf{m.d^2(x)/dt^2 = -kx \text{ --- --- } (ii)}$$

Eq (ii) is the differential equation for the simple harmonic motion.

3. Find a solution for equation (ii).

Answer:
Lets suppose we get x = (something). So if this (the solution) makes the LHS and RHS of the equation (ii) equal, then we may accept those variables as the solution of eq (ii).

If we put $x = 0$, then LHS = RHS. So $x = 0$ is a solution. Since $x = 0$ is not a useful solution, (trivial solution) we search for other solutions. We suppose a sinusoidal solution. $x = \sin(\theta)$, $x = \cos(\theta)$

Let us assume, $x = A\cos(kx - \omega t)$ is a solution of eq. (ii). So we use this in equation (ii) and see what happens.

$m.d^2(A\cos(kx - \omega t))/dt^2 = -kA\cos(kx - \omega t)$ --- --- (ii)

Differentiating once we get,

$(-)(-\omega)\, m.d(A\sin(kx - \omega t))/dt) = -kA\cos(kx - \omega t)$

Differentiating the LHS again we get

Or, $\omega.m.(-w)A\cos(kx - \omega t)$

Or, $-\omega^2.m.A\cos(kx - \omega t)$ -- - --- --- --- (iii)

Or $-\omega^2.m.(x)$

We know that $\omega^2 = K/m$, So, $-\omega^2.m = k$,

Using this relation in eq. (iii), we get,

- $K.x = -kx$

So the LHS = RHS

You can show, similarly, that ***x* = *Asin*(**kx - ωt***)*** is also a solution of equation (ii)

So today,

We've used $x = A\cos(kx-\omega t)$

We've found, $v = -\omega A\sin(kx-\omega t)$

We've found, $a = -\omega^2\cos(kx-\omega t)$